Satellite Selection in the Context of Network RTK for Limited Bandwidth Applications

Uttama Dutta, *RISE Research Institutes of Sweden*
Carsten Rieck, *RISE Research Institutes of Sweden*
Martin Håkansson, *Lantmäteriet*
Daniel Gerbeth, *German Aerospace Center (DLR)*
Samieh Alissa, *Lantmäteriet*
Stefan Nord, *RISE Research Institutes of Sweden*

BIOGRAPHY (IES)

Uttama Dutta received a Master's degree in Electronics and Telecommunication Engineering from Bengal Engineering and Science University, Kolkata, India in 2005. She has more than 13 years of work experience in research and academia with a focus on signal and image processing and navigational electronics. This includes positioning, localization and mitigation of interference and multipath in GNSS applications. Presently she works with RISE Measurement Science and Technology and is working on GNSS precise positioning applications in a NRTK scenario.

Carsten Rieck received the M.Sc. degree in electrical engineering in 2000, and a degree of Licentiate of engineering in 2016, both from Chalmers University of Technology, Gothenburg, Sweden. He is with the time and frequency group at RISE Research Institutes of Sweden since 2000 and is working with time metrological research.

Martin Håkansson is a GNSS researcher at Lantmäteriet, and has more than ten years of experience in precise GNSS development. He received his Ph.D. in Geodesy and Geoinformatics from KTH – Royal Institute of Technology in Sweden in February 2020, and he also has a MSc in Engineering Physics from Luleå University of Technology from 2008. His research interests revolve mainly around precise GNSS positioning, including the positioning techniques network-RTK and PPP, the effect of GNSS hardware biases on precise GNSS positioning, and precise GNSS positioning for future mass market applications.

Daniel Gerbeth received a Bachelor and Master's degree in Electrical Engineering and Information Technology from Karlsruhe Institute of Technology in 2014. After working in the field of sensor fusion and navigation aiding at Fraunhofer IOSB he joined the German Aerospace Center (DLR) in May 2015 where he is involved in the research on GBAS-based navigation and navigation for UAV applications.

Samieh Alissa is a PhD degree holder of electrical engineering with more than 22 years expertise in systems development, research, and teaching. His focus has been on GNSS applications with high accuracy, integrity and sensor fusion. Samieh worked during the last 10+ years in several aspects related to autonomous driving, functional safety, Network-RTK, SBAS and sensor fusion.

Stefan Nord received a Master's degree in Electrical Engineering from Chalmers University of Technology, Gothenburg, Sweden 1995. He has 12 years' experience working with Research and Advanced Engineering within the automotive industry. He joined RISE Measurement Science and Technology in 2016 and has since 2020 been working as a Research and Business Manager. His main focus is GNSS positioning and measurement technologies for automated transports.

ABSTRACT

The increasing number of modernized GNSS signals and the availability of multi-constellation receivers are crucial for improvements of both precision and robustness of GNSS based positioning. However, the abundance of GNSS observations is not always useable as applications, using differential positioning or other techniques, may have limitations with respect to computational resources or communication bandwidth for reference data, and therefore require a qualified selection of a subset of observations for positioning. This paper is based on the work conducted in the project PREParE SHIPS funded by the European Union Agency for the Space

Programme (EUSPA) on the specific application of Maritime Navigation using Network Real Time Kinematic (NRTK) and will focus on the satellite selection algorithms of the Prepare Ships dissemination solution. This study is motivated by data rate requirements and restrictions of the VDES dissemination solution developed in Prepare Ships. The restricted data rate for dissemination of RTK observations via VDES implies the need for a qualified pre-selection of satellite subsets to match the available bandwidth and the requirements of the positioning system. For this, multiple algorithms have been developed and tested in static and dynamic scenarios. Optimization techniques for height (for vertical position), two and three dimensions were examined. Different weighting schemes were used. During the evolution of the satellite selection study, it was concluded that it is necessary to retain satellites with the highest elevation as this will empirically improve integer ambiguity resolution for position fixing. Also fixing a minimum number of satellites for each constellation was required to enable a fair weightage to the different constellations used. Such algorithms should prove to be very useful for research on various Network RTK applications which require/prefer limited bandwidth such as for cadastral surveying and mapping, for airborne geo-referencing of aerial mapping data using Unmanned Aerial Vehicles (UAV) and on the road and sea for positioning and navigation of automated transport. Additionally, these algorithms could also be extended to consider satellite visibility in e.g. urban areas (i.e. urban canyons) by inclusion of true surface information for more robust GNSS positioning in automated transport applications [1]. This could either be for pre-evaluation or for dynamically considering spatial information. While this work is a part of PREParE SHIPS, it is also motivated by a more general applicability of the algorithms presented for other similar applications. RTK correction dissemination with limited bandwidth requirements is very promising for RTK research and therefore this study on optimized selection of satellite subsets is of vital importance and could tap multiple opportunities of huge potential such as those involving NRTK or combination of Precise Point Positioning with RTK.

I. INTRODUCTION

The use and application of Global Navigation Satellite Systems (GNSS) is steadily increasing as new constellations with improved signals are deployed, and the number of available satellites and services increase. The availability of observations from different constellations provides redundancy and potentially better positioning performance. It also allows to isolate and average out sources of errors and improves resiliency for critical applications. However, studies show that we now experience the problem of having more satellites/signals than may be tracked by a typical receiver in certain contexts like civil aviation [2]. Therefore, for certain applications a selection of an optimum subset of satellites is needed in order to meet for example requirements of computational complexity or transmission bandwidth. This paper studies how to optimally utilize all available resources in order to achieve satisfactory positioning performance for a given application from a NRTK perspective.

The topic of satellite selection in general has been covered widely in studies in the past, starting from the very early days in GPS in 1984 [3]. While some research focuses on specific applications like in civil aviation ([2, 4]), other studies theoretically analyze general bounds on dilution of precision (DOP) [5, 6]. In many cases, the goal of satellite selection is to find an optimal subset of all satellites in view to use for a certain application, where *optimal* can mean different things depending on the context. Previous research on satellite selection methods in the field of GBAS (Ground Based Augmentation System) was focused on mitigating problems with VDB (VHF Data Broadcast) capacity [4]. While some methods focus on optimizing the satellite geometry (i.e. DOP) alone, an important aspect especially in safety critical applications is to also consider residual range errors on the used satellite subset, as only both factors together determine the final positioning performance and reliability. In a similar manner, Walter et al. studied the applicability of satellite selection on SBAS and proposed a suitable algorithm [2]. General issues related to satellite selection, such as the influence of the number of selected satellites, satellite geometry and measurement error have also been investigated in [7]. Zhang and Zhang [8] in their work, gave a theorem about the bounds of Geometric Dilution of Precision (GDOP) values for any number of satellites. A lot of existing literature also focuses on assuming the best signal quality with highest elevation [9, 10] or minimizing Geometrical Dilution of Precision (GDOP) by maximizing polyhedron volumes or matrix determinants [3,11,12] without considering signal quality and therefore residual range errors in the selection process. Other more recent approaches using genetic algorithms [13] and artificial neural networks 14] also have also been studied earlier.

This paper is based on the work carried out in the project PREParE SHIPS funded by European GNSS Agency (GSA) on the specific application of Maritime Navigation using Network Real Time Kinematic. VDES [15] is proposed for dissemination of these observations and potentially restricts the available data rate. Therefore, there is a need for optimal selection of satellite/signal subsets from the set of the available GNSS provided by the NRTK service. The objective of the algorithms presented in this paper is to systematically choose a combination of specific signals, satellites or even constellations that optimizes RTK positioning performance as per guidelines mentioned in [16].

II. NETWORK RTK AND SWEPOS

The demand for Network Real Time Kinematic (NRTK) positioning services has been growing among the GNSS users for applications which require accurate and seamless kinematic position. For the optimum positioning services using current techniques, the average baseline length between NRTK reference stations has to be limited to between 30 km and 70 km and the reference station site must have good observation environment, should be stable, have access to power and have cabled or wireless internet or intranet. SWEPOS, the national CORS network of Sweden operated by Lantmäteriet is an example of NRTK. The present SWEPOS Network-RTK Service is based on the virtual reference station concept (VRS), with two-way mobile network communication between the control center and the RTK users. The network computing center is located in Lantmäteriet (Gävle) and it generally determines various errors (including atmospheric errors, clock errors, and local multipath) by fixing the ambiguities of the baselines within the network, simulates the position of the VRS by geometrically displacing the data of the reference station closest to the rover, interpolates the network errors at the VRS location using linear or more sophisticated models, and finally transmits the corrections to the rover in real-time. The present SWEPOS infrastructure consists of approximately 450 permanent GNSS reference stations located as shown in Figure 1.

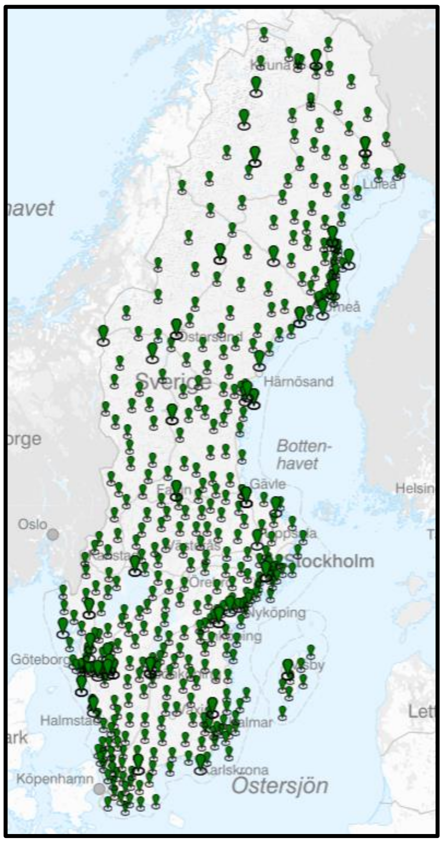


Figure 1: SWEPOS reference stations.

Figure 2: Class A station Leksand.

Figure 3: Storuman, a class B station.

In general, SWEPOS has two types of reference stations: Class A stations and Class B stations. The Class A stations (21 stations) have the best long-term coordinate stability because the GNSS antennas are mounted on insulated concrete pillars or truss masts with fixed anchorage in crack-free bedrock. Other equipment at the stations is installed in some type of technology shed. There is good redundancy for GNSS measurement, data communication and power supply, i.e. reserve capacity that can be activated in the event of a problem. The Class A stations are also used to monitor the coordinate stability of the Class B stations. Figure 2 shows an example of the class A station. The Class B stations are densifying the network of Class A stations in the expansion of the SWEPOS network that is being done to increase the capacity for real-time measurement. Figure 3 shows an example of the class B station.

Dissemination of reference data is traditionally organized peer-to-peer by generating individual data streams per user utilizing IP networks. Terrestrial VDES is the proposed communication solution in PREParE SHIPS and provides 100 kHz communication channels in the VHF band at around 162 MHz. VDES inherited time division access from AIS and for the purpose of the distribution of GNSS RTK reference data, areal broadcast using one slot per second is suggested. This provides a net-bandwidth of 650 bytes/s using link ID 19. VHF communication is line of sight restricted and typical ranges are in the order of 10 nautical miles which suggest a few virtual references to be disseminated per base station. This strict bandwidth limitation is the main driver to optimize the reference data that is broadcasted to the user.

III. METHODOLOGY

The satellite selection subset we use is based on an **S**-matrix as a basis for the selection criteria. The satellite specific entries in this S-matrix (describing basically the projection from range into position domain) are weighted using another matrix called **P** to finally form the 'S-measures' per satellite. These measures to perform the initial selection are similar to the work carried out by Gerbeth et al. [17] for GBAS and Walter et al. [2] for SBAS except for the fact that it is modified to suit our application of maritime navigation using NRTK. The matrices **Q** (the co-variance matrix) and **S** are defined as,

$$\mathbf{Q} = (\mathbf{G^T} \cdot \mathbf{W} \cdot \mathbf{G})^{-1} \quad \text{and} \qquad \mathbf{S} = \mathbf{Q} \cdot \mathbf{G^T} \cdot \mathbf{W} \tag{1}$$

Where each row of **G**, known as $\mathbf{G_i}$ for the i[th] satellite with azimuth θ_i and elevation $\emptyset_i$ is defined as,

$$\mathbf{G_i} = [-\cos(\emptyset_i)\cos(\theta_i) \qquad -\cos(\emptyset_i)\sin(\theta_i) \qquad -sin(\emptyset_i) \qquad 1 \;] \tag{2}$$

We define the measures for initial selection for vertical, two- and three-dimensional optimization as,

$$S_{meas,i}(vertical) = \frac{S_{3,i}^2}{P_{i,i}} \tag{3}$$

$$S_{meas,i}(2D) = \frac{S_{1,i}^2 + S_{2,i}^2}{P_{i,i}} \tag{4}$$

$$S_{meas,i}(3D) = \frac{S_{1,i}^2 + S_{2,i}^2 + S_{3,i}^2}{P_{i,i}} \tag{5}$$

Where $\quad \mathbf{P} = \mathbf{W} - \mathbf{W} \cdot \mathbf{G} \cdot \mathbf{S}$ (6)

Equations (1) through (6) assume **W** to be a weighting matrix about which we discuss below. Though one might argue that two-dimensional optimization should suffice for an application such as the sea surface it is important to understand that the sea level is all but homogenous and therefore the draught of the ship and the under-keel clearance are of utmost importance.

In the work carried out in [4] for GBAS it was observed that the value of the projection factor from the pseudo-range into the vertical position of a satellite inside a constellation exhibited a strong influence on the satellite being a part of an ideal subset of satellites for vertical position accuracy. The same applies to 2D and 3D positioning with the combined projection factors. However, though satellite selection based only on elevation angle may be good from a tracking robustness point of view, but it does not necessarily lead to the best availability [2].

The algorithm for satellite subset selection in this work provides flexibility for different streams of reference data that can be tailored for different application needs. The most sophisticated options intend to select a balanced set of satellites with optimization for either vertical, two- or three-dimensional positioning (based on equations (3), (4) and (5)), using either User Equivalent Range Error (UERE), Signal to Noise ratio (SNR) or elevation-based weighting schemes.

Satellite Weighting Matrix Based on UERE, SNR and Elevation

The user equivalent range error (UERE) was computed based on assumptions and calculations presented by Shuai et al. [18] as,

$$\sigma_{UERE}^2 = \sigma_{URE}^2 + \sigma_{Iono}^2 + \sigma_{Trop}^2 + \sigma_{MP}^2 + \sigma_{noise}^2 \tag{7}$$

Where σ_{URE}^2 is the variance of user ranging error, σ_{Iono}^2 is the variance of ionospheric delay, σ_{Trop}^2 is the variance of tropospheric delay, σ_{MP}^2 is the variance of multipath error and σ_{noise}^2 is the variance of noise error. However, [18] focuses on single point positioning. While adapting to differential techniques such as NRTK it was important to understand that the contribution to UERE of the carrier phase residuals would be primarily by multipath and troposphere. These have been modelled based on elevation angle ($\emptyset$) as in [18] for an initial guess estimate of UERE that could be involved in NRTK scenario,

$$\sigma_{Trop}^2 = \frac{0.12012}{\sqrt{0.002001 + sin^2(\emptyset)}} \tag{8}$$

$$\sigma_{MP}^2 = 0.5 + 1.64e^{-(\frac{\emptyset}{14.5°})} \tag{9}$$

The weight matrix based on UERE values for the i[th] satellite can then be estimated as,

$$\boldsymbol{W} = \begin{bmatrix} \frac{1}{\sigma_{1,UERE}^2} & 0 & \dots & 0 \\ 0 & \frac{1}{\sigma_{2,UERE}^2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \frac{1}{\sigma_{i,UERE}^2} \end{bmatrix} \tag{10}$$

Based on the work done in [19], the satellite weight matrix based on elevation angle for the i[th] satellite ($\emptyset_i$) was estimated as,

$$\boldsymbol{W} = \begin{bmatrix} sin^2(\emptyset_1) & 0 & \dots & 0 \\ 0 & sin^2(\emptyset_2) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & sin^2(\emptyset_i) \end{bmatrix} \tag{11}$$

Based on the work done in [20], the satellite weight matrix based on Signal to noise ratio derived from the C/A data for the i[th] satellite (SNR_i) was estimated as,

$$\boldsymbol{W} = \begin{bmatrix} 10^{0.1SNR_1} & 0 & \dots & 0 \\ 0 & 10^{0.1SNR_2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 10^{0.1SNR_i} \end{bmatrix} \tag{12}$$

Based on the work done in [20], the satellite weight matrix based on a combination of elevation angle and the Signal to noise ratio derived from the C/A data for the i[th] satellite ($\emptyset_i$ and SNR_i) was estimated as,

$$\boldsymbol{W} = \begin{bmatrix} sin^2(\emptyset_1) * 10^{0.1SNR_1} & 0 & \dots & 0 \\ 0 & sin^2(\emptyset_2) * 10^{0.1SNR_2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & sin^2(\emptyset_i) * 10^{0.1SNR_i} \end{bmatrix} \tag{13}$$

A case for unweighted geometry was also considered where the weight matrix in equation (1) is simply an identity matrix.

A clear outline of how the algorithms work is presented in Figure (4). Vertical Dilution of Precision (VDOP), Horizontal Dilution of Precision (HDOP) and Position Dilution of Precision (PDOP) derived from square root of sum of corresponding diagonal elements of **Q** were used as measures for optimization.

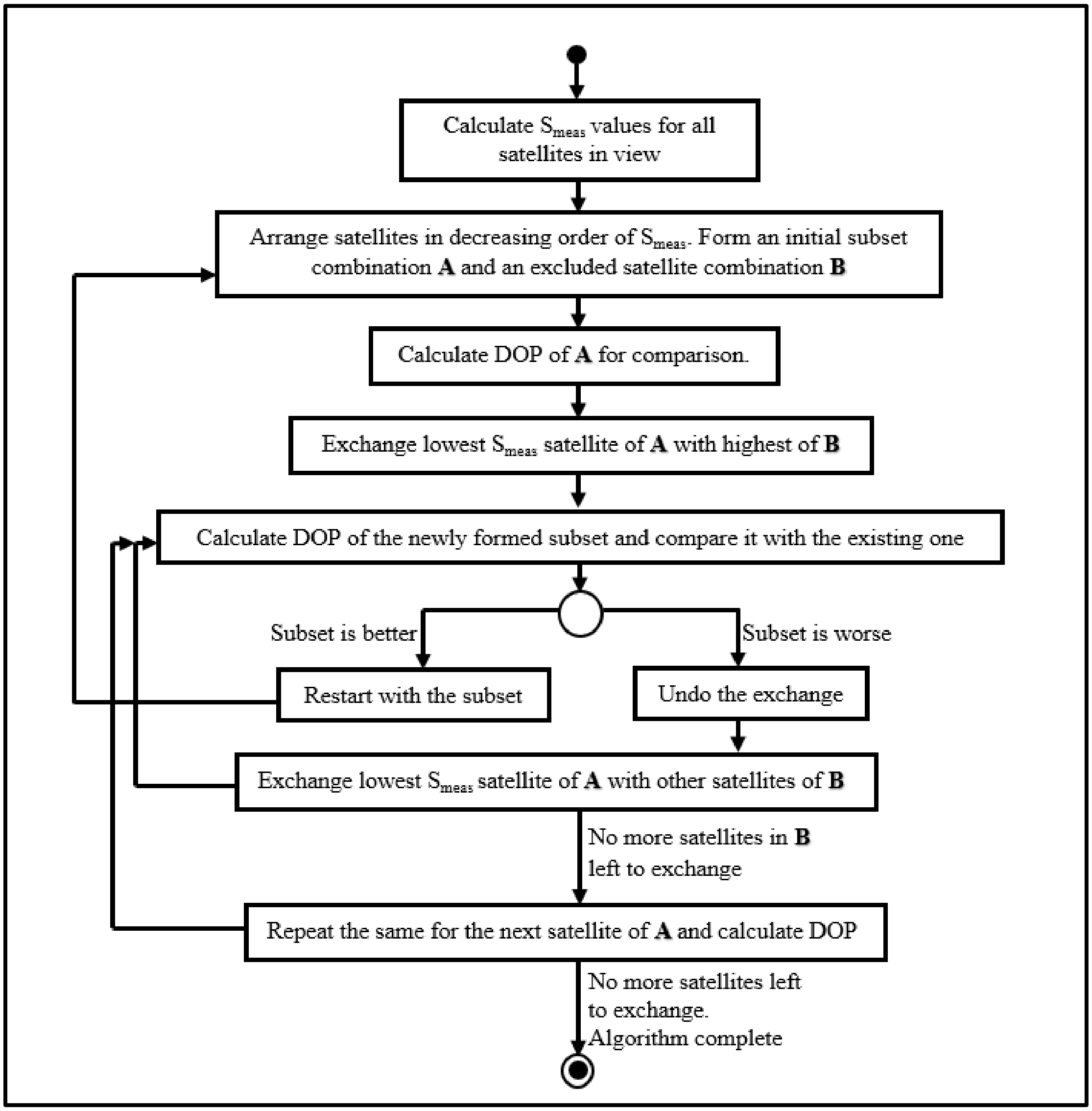


Figure 4: Flow chart for satellite selection algorithm

Experimental Setup

To enable testing and support of algorithm development an experimental test environment was created. As code base served a RTCM3 to RINEX converter of the early BNC project [21]. This software was extended to time-tag RTCM3 streams, raw data save to file and a timely replay functionality. The selection algorithms were integrated together with a RTKLib based processing engine [22]. Message manipulation was implemented on MSM messages only. The concept is expandable to other in-line manipulations such as clock corrections or harmonization of carrier phase ambiguities across a network of base stations. The setup allows rapid scripted replay for testing different selection scenarios and RTK processing parameters. Selection can also be done in real-time and provides low latency manipulated RTCM streams to RTK engines in GNSS receivers using IP or serial interfaces.

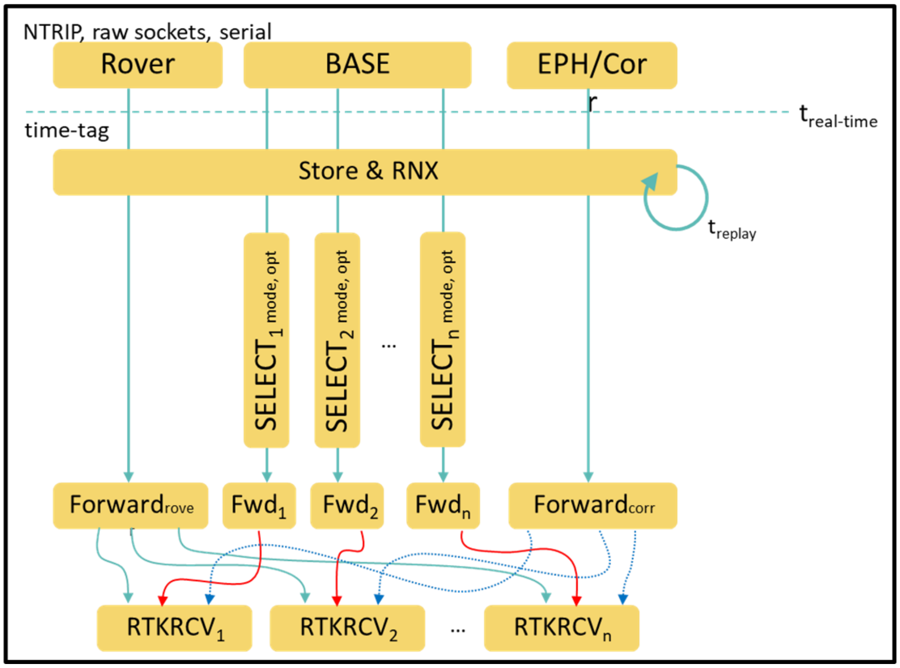


Figure 5: Experimental test setup for processing with RTKLib. Initially intended as a RTCM manipulating and relay function the software was extended to save RTCM raw data streams and enable rapid replay in different combinations.

IV. RESULTS

Validation of the Algorithms

The satellite selection algorithms were tested using the open source RTKLib version 2.4.3- b33 and b34 (demo5) using different number of satellites for GPS and Galileo constellation. Sky plots from RTKNavi for a typical static trial example are shown in Figure (6). This example validates the algorithms by using ten concurrent sessions of RTKNAVI streaming corrections in real time simultaneously from the satellite selection software as the reference, and Ublox F9P as the rover used in NRTK mode. This test has been carried out for a GPS only testcase as an example with only four satellites taken into consideration. There is also an algorithm (at the end of the figure) designed to retain satellites above 30 degrees of elevation and does not take geometry into consideration.

Unweighted Height Optimization
Solution: FLOAT
X: 3328973.693 m
Y: 761877.652 m
Z: 5369051.752 m
Unweighted Two dimensional optimization
Solution: FIX
X: 3328974.021 m
Y: 761878.486 m
Z: 5369051.499 m
Unweighted Three dimensional optimization
Solution: FIX
X: 3328973.901 m
Y: 761878.462 m
Z: 5369051.349 m
Elevation weighted Height Optimization
Solution: FIX
X: 3328973.928 m
Y: 761878.502 m
Z: 5369051.334 m
Elevation weighted Two dimensional optimization
Solution: FIX
X: 3328974.005 m
Y: 761878.481 m
Z: 5369051.475 m
Elevation weighted Three dimensional optimization
Solution: FIX
X: 3328973.928 m
Y: 761878.502 m
Z: 5369051.334 m
UERE weighted Height Optimization
Solution: FLOAT
X: 3328973.469 m
Y: 761878.958 m
Z: 5369049.614 m
UERE weighted Two dimensional optimization
Solution: FIX
X: 3328974.021 m
Y: 761878.486 m
Z: 5369051.499 m
UERE weighted Three dimensional optimization
Solution: FLOAT
X: 3328973.488 m
Y: 761878.912 m
Z: 5369049.673 m

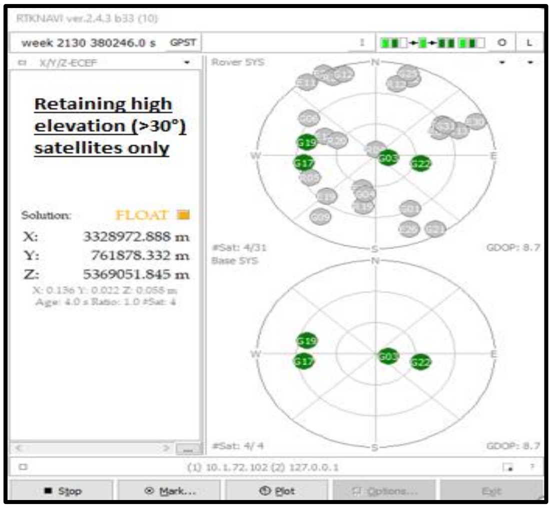


Figure 6: Sky plots of ten parallel runs of RTKNAVI of the RTKLib with reference data provided by Satellite selection algorithms for static trials on 5th Nov2020. Each window indicates the stream in corrections for a specific algorithm as mentioned in the window.

Tests and Results

The algorithms with different weighting and optimization schemes mentioned earlier in equations (1)-(13) were tested for static and dynamic positioning using the test setup shown in Figure (5). Also, the tests use constellations with equal number of satellites, so that fair weightage is given to each constellation. The results presented in this paper focus on selection algorithms that use 5 satellites each from the GPS and Galileo constellation. However, other constellations and different sizes of satellite selection subsets have also been tested. The data in the plots are sampled at 1 second intervals. The error differences from one case to another was marginal and therefore system level performance criteria such as Time to First Fix (TTFF) and percentage of position fix were also used for the comparative analysis.

Static test results

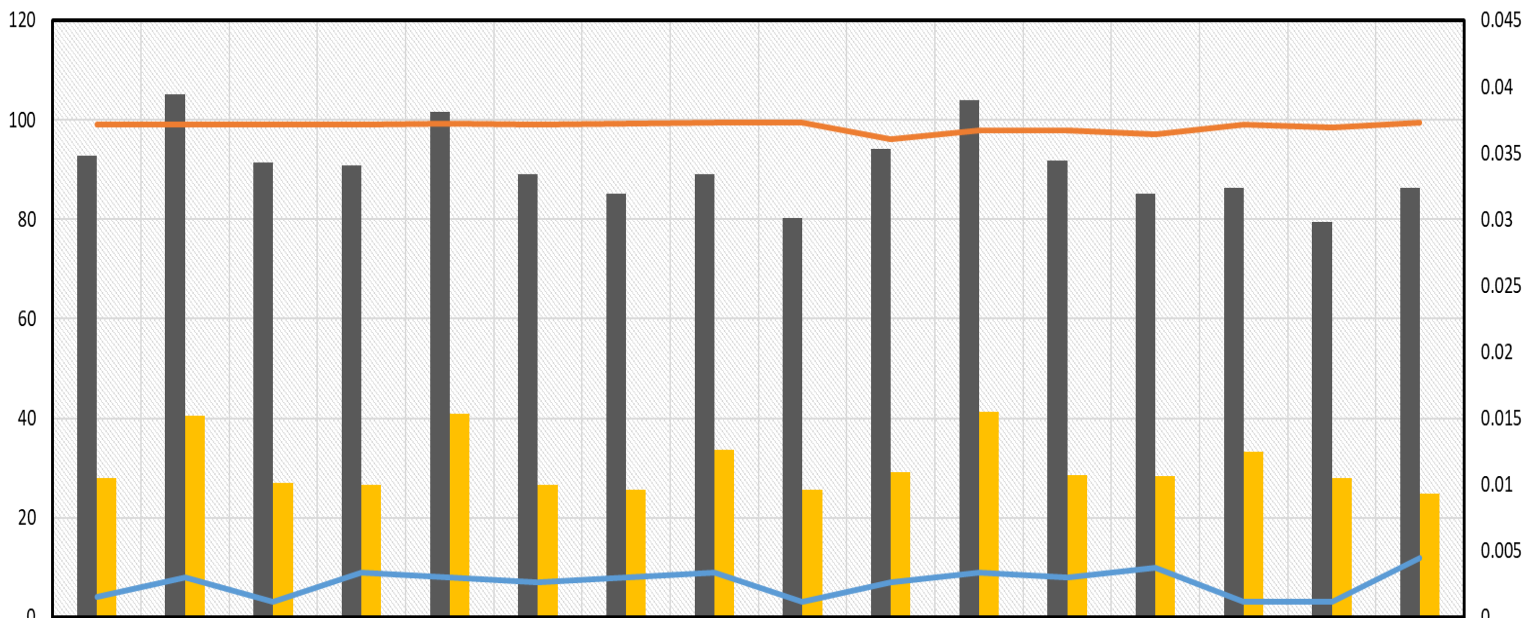
For static positioning, a set of Virtual Reference Stations (VRSs) from SWEPOS were used to offer true ground conditions when testing the algorithms. One of the VRS is used as a rover and another as the base for the tests carried out. The data streams are composed of MSM4 messages, excluding Glonass, and reoccurring meta information. A reference/baseline case with no selection is generated for comparison in all the static tests. This means that for this case, RTKLib is free to choose any number of satellites that are visible to the receiver for computing the positioning solution. A group of 15 algorithms have been generated as of now, and outputs from all these and a 16th baseline case are numbered and named as outlined in Figure (7). These algorithm numbers and names are used for the plots in this section.

Satellite selection Algorithm	
Number	Name
H	Unweighted geometry optimization for H
2	Unweighted geometry optimization for 2D
3	Unweighted geometry optimization for 3D
H_UERE	UERE weighted geometry optimization fo H
2_UERE	UERE weighted geometry optimization for 2D
3_UERE	UERE weighted geometry optimization for 3D
H_El	Elevation weighted geometry optimization for H
2_El	Elevation weighted geometry optimization for 2D
3_El	Elevation weighted geometry optimization for 3D
H_SNR	SNR weighted geometry optimization for H
2_SNR	SNR weighted geometry optimization for 2D
3_SNR	SNR weighted geometry optimization for 3D
H_comb	SNR and elevation weighted geometry optimization for H
2_comb	SNR and elevation weighted geometry optimization for 2D
3_comb	SNR and elevation weighted geometry optimization for 3D
baseline	Baseline with no selection algorithm

Figure 7: Selection algorithm names and numbers

The plot in Figure (8) shows statistics for position errors for GPS and GAL for various optimization algorithms tested simultaneously on 3rd Sept 2021 for a duration of 1 day, 4 hours, 42 minutes approximately. There is a 12 km baseline distance between the Virtual Reference Stations. All the algorithms for weighting namely, UERE, elevation, SNR, combination (SNR and elevation) and the unweighted case have been tested. One can see in this figure, that many of the selection algorithms (especially the combination and elevation weighting cases) show better performance than the baseline case. This is not only in terms of error statistics, but also in terms of TTFF and percentage of position fix.

Figure (9) plots for a smaller duration the number of satellites actually used for the position computation by cases 'baseline' and 'H_comb' of Figure (8). The baseline case is free to choose any number of satellites from those viewed by the receiver and uses 15 to 19 satellites in this case which is definitely larger than the number of satellites used by case 'H_comb' which is restricted to choosing only 10 (5 from each constellation). The reduced number of satellites from the selection algorithm implies fewer bytes (in this case about 25% bytes lesser) for correction dissemination via VDES. This definitely demonstrates that reduced number of carefully chosen observations from specific satellites is sufficient for precise positioning even on a 12 km baseline.



	H	2	3	H_UERE	2_UERE	3_UERE	H_El	2_El	3_El	H_SNR	2_SNR	3_SNR	H_comb	2_comb	3_comb	baseline
Mean pos error(m)	0.0348	0.0394	0.0343	0.0341	0.0381	0.0334	0.0319	0.0334	0.0301	0.0353	0.039	0.0344	0.0319	0.0324	0.0298	0.0324
Deviations of pos error(m)	0.0105	0.0152	0.0101	0.01	0.0153	0.01	0.0096	0.0126	0.0096	0.0109	0.0155	0.0107	0.0106	0.0125	0.0105	0.0093
Time to First fix(s)	4	8	3	9	8	7	8	9	3	7	9	8	10	3	3	12
Percentage of Pos fix	99.031	99.111	99.074	99.1	99.217	99.117	99.337	99.374	99.373	96.057	97.898	97.839	97.061	98.969	98.478	99.493

Mean pos error(m) Deviations of pos error(m) Time to First fix(s) Percentage of Pos fix

Figure 8: Statistics for position errors for static tests using GPS and GAL (5 satellites each)-Optimization algorithms tested on 3rd Sept 2021 (Baseline distance of 12km between Rover and Base)

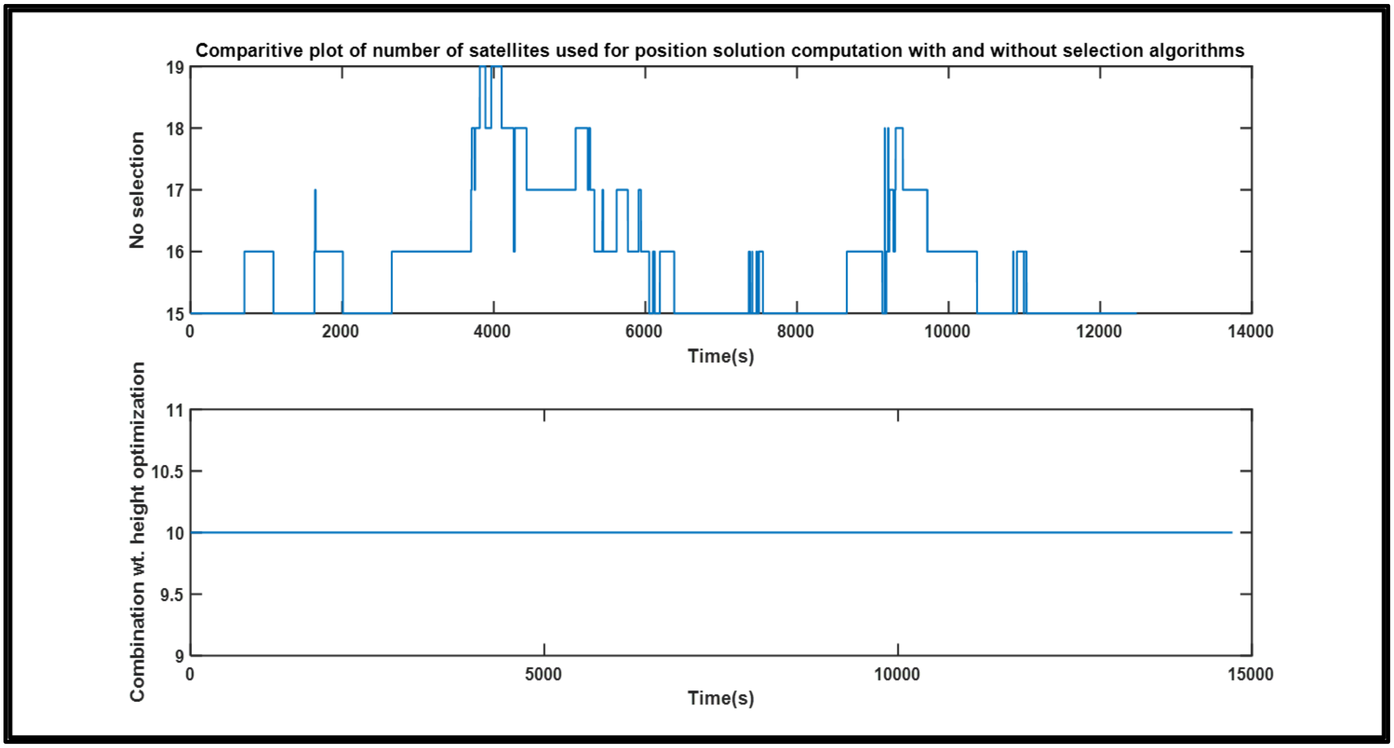
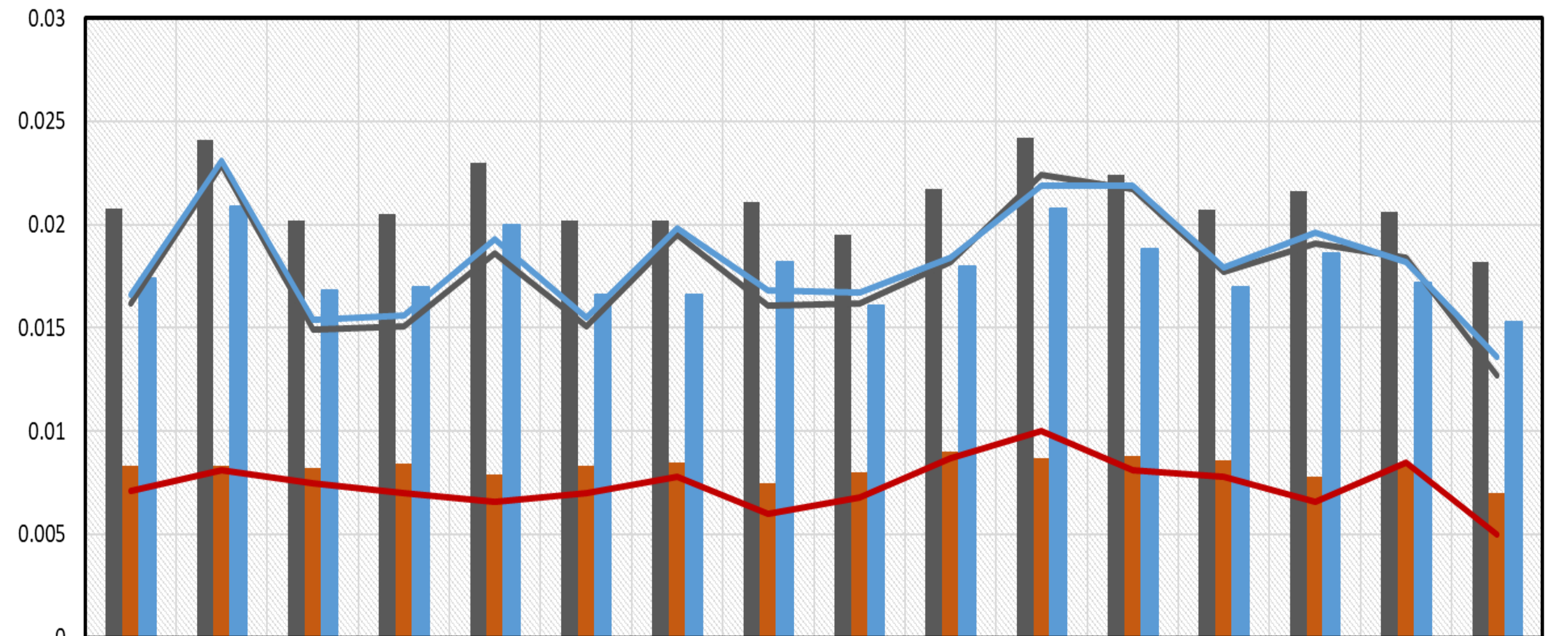


Figure 9: Comparative plot of number of satellites used for positioning solution tested for height optimization algorithms on 3rd Sept 2021.

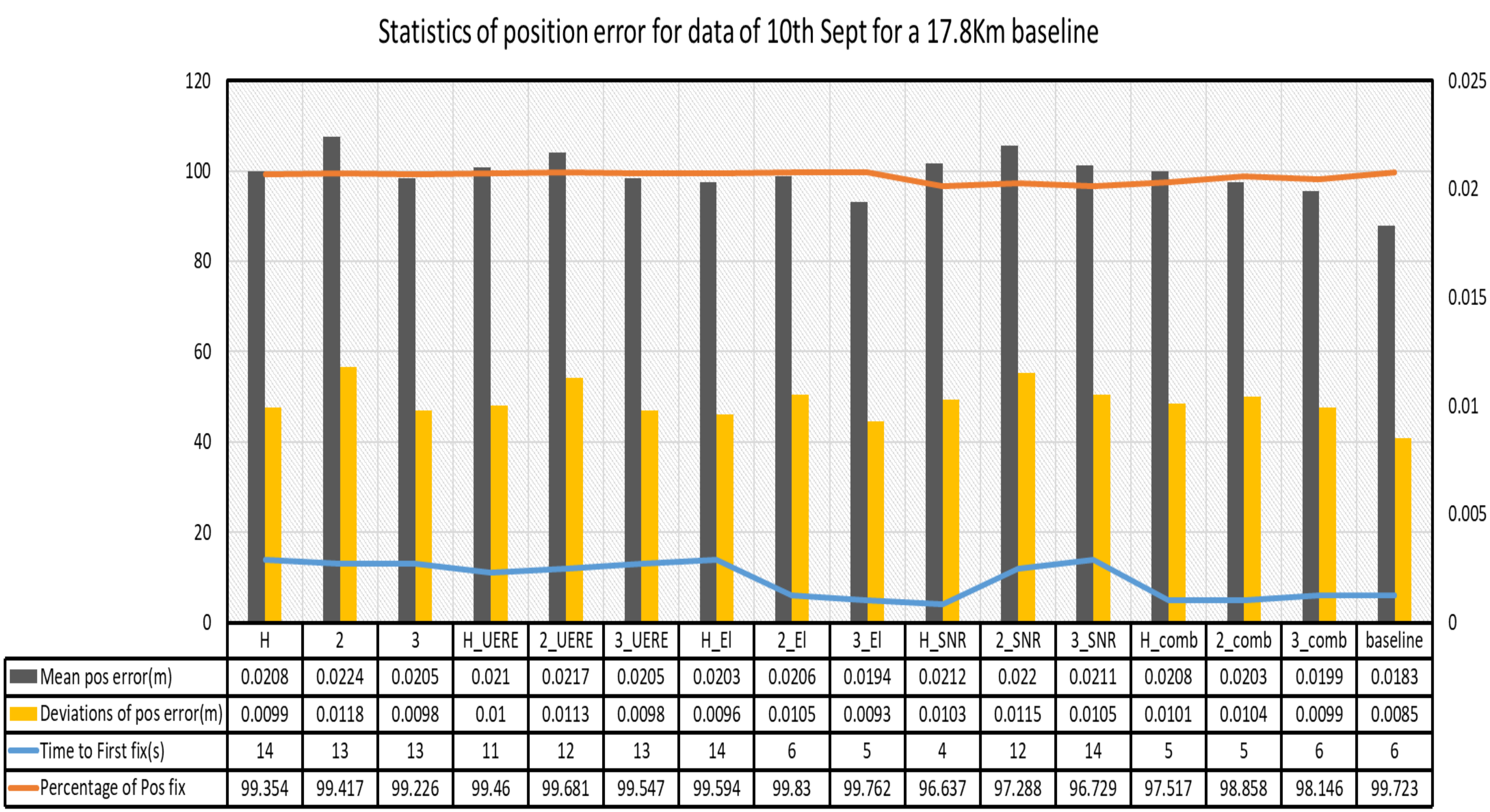
It was important to investigate how the optimization algorithms perform for larger baseline length solutions. The plot in Figure (10) shows the UTM (Universal Transverse Mercator) statistics for errors in height, two and three dimensions for GPS and GAL for all the optimization algorithms shown in Figure (7) used simultaneously. This test was carried out during 10th-16th Sept 2021 for a duration of approximately 5days and 22 hours and is for a baseline distance of 17.8 km between Rover and Base. From the plot we can see that the given optimization algorithms have improved error statistics in the respective cases when compared to the cases with other algorithms. For example, the 3D optimization algorithm for elevation weighting (3_El), shows improved error statistics for 3D error compared to the 3D error statistics of the 2D or height optimization algorithms. One can easily verify this for the other error statistics of the other cases. Also, in some cases the height optimization performance is comparable with the 3D optimization performance, which is expected. This demonstrates the potential of the optimization algorithms used.

The plot in Figure (11) shows the statistics for position errors for GPS and GAL for the test of Figure(10) and was meant to understand possibilities for the large base line length in this case of 17.8km. It can be seen that performances of all the algorithms are nearly similar to the baseline case with very high percentages of position fix. It was also observed that, some of the algorithms which consider elevation as a weighting criterion alone or with a combination of SNR perform better than the other selection algorithms in most cases.



	H	2	3	H_UERE	2_UERE	3_UERE	H_El	2_El	3_El	H_SNR	2_SNR	3_SNR	H_comb	2_comb	3_comb	baseline
Mean3D (m)	0.0208	0.0241	0.0202	0.0205	0.023	0.0202	0.0202	0.0211	0.0195	0.0217	0.0242	0.0224	0.0207	0.0216	0.0206	0.0182
Mean2D (m)	0.0083	0.0083	0.0082	0.0084	0.0079	0.0083	0.0085	0.0075	0.008	0.009	0.0087	0.0088	0.0086	0.0078	0.0083	0.007
Mean_H (m)	0.0174	0.0209	0.0168	0.017	0.02	0.0166	0.0166	0.0182	0.0161	0.018	0.0208	0.0188	0.017	0.0186	0.0172	0.0153
stddev3D (m)	0.0162	0.0229	0.0149	0.0151	0.0186	0.0151	0.0195	0.0161	0.0162	0.0182	0.0224	0.0217	0.0177	0.0191	0.0184	0.0127
StdDev 2D (m)	0.0071	0.0081	0.0075	0.007	0.0066	0.007	0.0078	0.006	0.0068	0.0087	0.01	0.0081	0.0078	0.0066	0.0085	0.005
Stddev H (m)	0.0166	0.0231	0.0154	0.0156	0.0193	0.0155	0.0198	0.0168	0.0167	0.0184	0.0219	0.0219	0.0179	0.0196	0.0182	0.0136

Figure 10: GPS and Galileo satellites tested for optimization algorithms on 10th Sept 2021



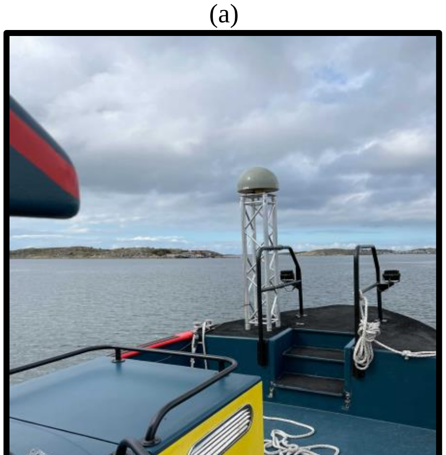
	H	2	3	H_UERE	2_UERE	3_UERE	H_El	2_El	3_El	H_SNR	2_SNR	3_SNR	H_comb	2_comb	3_comb	baseline
Mean pos error(m)	0.0208	0.0224	0.0205	0.021	0.0217	0.0205	0.0203	0.0206	0.0194	0.0212	0.022	0.0211	0.0208	0.0203	0.0199	0.0183
Deviations of pos error(m)	0.0099	0.0118	0.0098	0.01	0.0113	0.0098	0.0096	0.0105	0.0093	0.0103	0.0115	0.0105	0.0101	0.0104	0.0099	0.0085
Time to First fix(s)	14	13	13	11	12	13	14	6	5	4	12	14	5	5	6	6
Percentage of Pos fix	99.354	99.417	99.226	99.46	99.681	99.547	99.594	99.83	99.762	96.637	97.288	96.729	97.517	98.858	98.146	99.723

Figure 11: The statistics for position errors for GPS and GAL (5 satellites each)- Optimization algorithms tested on 10th Sept 2021 (Baseline distance of 17.8km between Rover and Base)

Dynamic test results

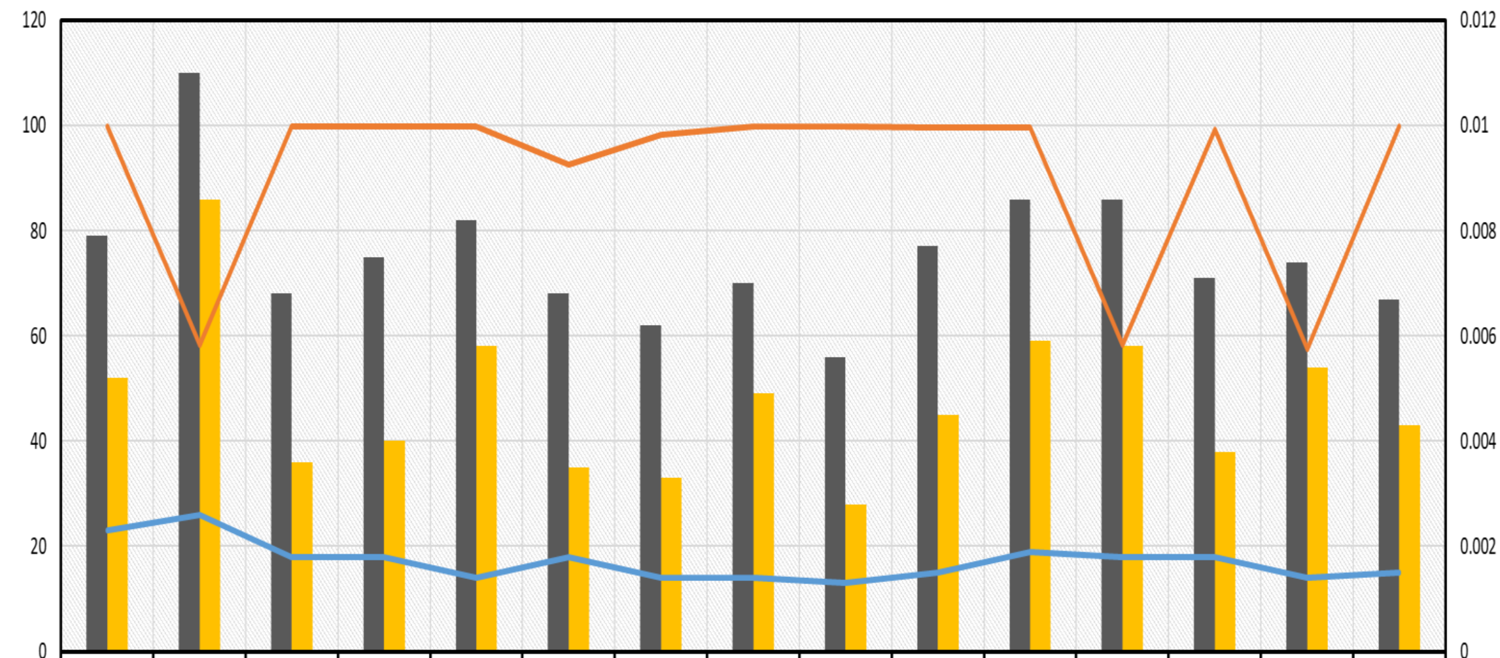
A set of Trimble receivers and antennas mounted on an a search and rescue type of vessel (SSRS) were used for dynamic trials carried out on 14th sept 2021. The Trimble receivers acted as rovers and the VRSs used as base for the tests of Figures (10) and (11) acted as base stations to relay of RTK corrections. Figure (12) shows the ground track for a single trial and the antenna installation for the dynamic tests. The total test duration is around 4.5 hours. The average baseline length between the moving rover and the base station was about 13.7 km with a maximum and a minimum baseline length of about 17.8 and 4.5 km, respectively.

(a)



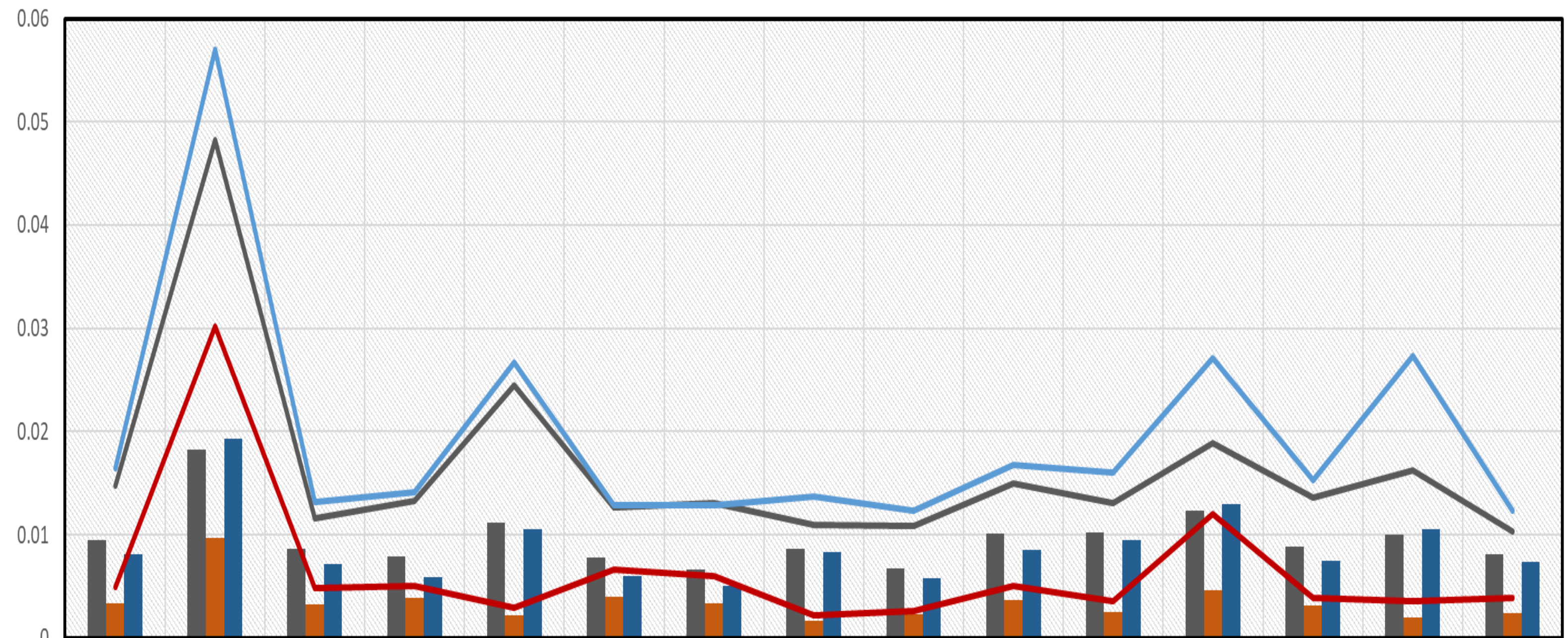
(b)

Figure 12: (a) Ground track and (b) Antenna installation on the SRS boat for dynamic sea trials carried out on 14th Sept 2021.



	H	2	3	H_UERE	2_UERE	3_UERE	H_El	2_El	3_El	H_SNR	2_SNR	3_SNR	H_comb	2_comb	3_comb
Mean pos error(m)	0.0079	0.011	0.0068	0.0075	0.0082	0.0068	0.0062	0.007	0.0056	0.0077	0.0086	0.0086	0.0071	0.0074	0.0067
Deviations of pos error(m)	0.0052	0.0086	0.0036	0.004	0.0058	0.0035	0.0033	0.0049	0.0028	0.0045	0.0059	0.0058	0.0038	0.0054	0.0043
Time to First fix(s)	23	26	18	18	14	18	14	14	13	15	19	18	18	14	15
Percentage of Pos fix	99.771	58.287	99.815	99.828	99.828	92.528	98.275	99.847	99.854	99.573	99.599	58.309	99.198	57.515	99.72

Figure 13: Statistics for position errors for dynamic tests using GPS and GAL (5 satellites each) -Optimization algorithms tested on 14th Sept 2021 (Average baseline distance of 13.7km between Rover and Base)



	H	2	3	H_UERE	2_UERE	3_UERE	H_El	2_El	3_El	H_SNR	2_SNR	3_SNR	H_comb	2_comb	3_comb
Mean3D (m)	0.0095	0.0183	0.0086	0.0079	0.0112	0.0078	0.0066	0.0087	0.0068	0.0101	0.0102	0.0123	0.0089	0.01	0.0081
Mean2D (m)	0.0034	0.0097	0.0033	0.0039	0.0022	0.004	0.0034	0.0017	0.0023	0.0037	0.0025	0.0046	0.0032	0.002	0.0024
Mean_H (m)	0.0081	0.0193	0.0072	0.0059	0.0105	0.006	0.0051	0.0083	0.0058	0.0085	0.0095	0.013	0.0075	0.0105	0.0074
stddev3D (m)	0.0148	0.0482	0.0116	0.0133	0.0245	0.0127	0.0131	0.011	0.0109	0.015	0.0131	0.0189	0.0136	0.0162	0.0103
StdDev 2D (m)	0.005	0.0302	0.0048	0.0051	0.003	0.0066	0.006	0.0022	0.0026	0.0051	0.0036	0.012	0.0039	0.0036	0.0039
Stddev H (m)	0.0165	0.057	0.0132	0.0141	0.0267	0.0129	0.0129	0.0137	0.0123	0.0168	0.016	0.0271	0.0153	0.0273	0.0123

Figure 14: UTM statistics for dynamic tests using GPS and GAL (5 satellites each)-Optimization algorithms tested on 14th Sept 2021 (Average baseline distance of 13.7km between Rover and Base)

Figure (13) shows the plot for the statistics of position errors and Figure (14) shows the UTM error statistics for the data collected during the dynamic trials using GPS and GAL constellations. These errors were computed using the baseline/reference case mentioned in Figure (7) as ground truth. The algorithms show comparable positioning performance in most cases. In particular the elevation weighting algorithms seem to show satisfactory positioning performance. The TTFF and the percentage of position fix for the baseline case without any selection was computed as 13 seconds and 97.443% respectively.

V. CONCLUSIONS AND SIGNIFICANCE OF WORK.

From the results shown, it can be concluded that for the given number of selected satellites (10 used in all examples), the selection algorithms provide positioning performances which are comparable in most cases to the baseline or no selection case. Therefore, a reduced number of carefully chosen observations from specific satellites is sufficient for precise centimeter level positioning. Some of the algorithms (such as those considering elevation as a weighting criteria) perform better than the others but in the end the differences in the performances are marginal.

RTK differential positioning exploits the integer nature of the carrier phase by resolving the phase ambiguity. Algorithms for this are often similar, but strategies may vary between different implementations. It can be assumed that most common solutions expect a contingency of a common set of satellites. During the evolution of the satellite selection study it was for instance concluded that it is needed to implement algorithms for retaining the satellite with the highest elevation, and to fix a minimum number of satellites for each constellation. This empirically improves integer ambiguity resolution for position fixing and enables a fair weightage to the different constellations used. As part of future work, it is of interest for us to study and develop new algorithms such as those meant for retaining high elevation satellites between epochs, more robust forms of SNR weighting algorithms etc.

Development and testing of satellite selection algorithms were primarily supporting the development of Lantmäteriet Adjustment Solution (LAS) in the context of the Prepare Ships project but was also important for understanding the consequences of bandwidth limitations for VHF dissemination from a user perspective. This research is among the first of its kind in the application of Network RTK in maritime and similar environment and resulted in adapted algorithms. However, the research and the achieved results extend beyond maritime applications and can be used for any general application for satellite selection with a Network RTK perspective.

ACKNOWLEDGMENTS

This work is carried out as part of the PREParE SHIPS project [23] and has been supported by funding from the European Union Agency for the Space Programme under the European Union’s Horizon 2020 research and innovation programme under grant agreement No 870239. We also acknowledge the RTKLib project initiated by Tomoji Takasu and contributed to by others, notably Tim Everett of rtkexploer.com, Dirk Stöcker for the rtcm3torinex tool that was heavily abused.

REFERENCES

1. Han, JY., Li, PH. Utilizing 3-D topographical information for the quality assessment of a satellite surveying. *Appl Geomat 2, 21–32 (2010).* https://doi.org/10.1007/s12518-010-0016-y
2. Walter T, Blanch J, Kropp V. Satellite selection for multi-constellation SBAS. *Proceedings of the 29th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2016); September 2016; Portland, OR*:1350-1359.
3. Masahiko Kihara and Tsuyoshi Okada. “A Satellite Selection Method and Accuracy for the Global Positioning System”. *In: NAVIGATION, Journal of the Institute of Navigation,* Volume 31, Number 1. 1984.
4. Gerbeth D, Felux M, Circiu M-C, Caamano M. Optimized selection of satellite subsets for a multi-constellation GBAS. *Proceedings of the 2016 International Technical Meeting of The Institute of Navigation; January 2016; Monterey, CA*:360-367.
5. Swaszek, Peter F., Richard J. Hartnett, and Kelly C. Seals. "Lower bounds on DOP." The Journal of Navigation 70.5 (2017): 1041-1061.

6. Swaszek, Peter F., et al. "Tighter GDOP Bounds and Their Use in Satellite Subset Selection." Proceedings of the 32nd International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2019). 2019.
7. Tao, A. L., & Jan, S. S. (2016). Optimal navigation with multi-constellation GNSS: A satellite selection algorithm. *Proceedings of the 29th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2016); September 2016; Portland, OR*: 128-139, Vol.1. Institute of Navigation. https://doi.org/10.33012/2016.14725
8. Miaoyan Zhang and Jun Zhang. "A Fast Satellite Selection Algorithm: Beyond Four Satellites". *In: IEEE Journal of Selected Topics in Signal Processing 3.5 (Oct. 2009)*, p. 8. ISSN: 1932-4553. DOI: 10.1109/jstsp.2009.2028381. URL: http://dx.doi.org/10.1109/JSTSP.2009.2028381.
9. Dan Song et al. "An Algorithm of Selecting more than Four Satellites from GNSS". *In: International Conference on Advanced Computer Science and Electronics Information (ICACSEI 2013)*. 2013.
10. Chan-Woo Park and Jonathan P. How. "METHOD AND APPARATUS FOR SELECTING OPTIMAL SATELITTES IN GLOBAL POSITIONING SYSTEM". Pat. US 6,727,850 B2. 2004.
11. Nuria Blanco-Delgado and Fernando D. Nunes. "Satellite Selection Method for Multi-Constellation GNSS Using Convex Geometry". *In: IEEE Transactions on Vehicular Technology 59.9 (Nov. 2010)*. ISSN: 1939-9359. DOI: 10.1109/tvt. 2010.2072939. URL: http://dx.doi.org/10.1109/TVT.2010.2072939.
12. Nuria Blanco-Delgado, Fernando D. Nunes, and Gonzalo Seco-Granados. "Relation between GDOP and the Geometry of the Satellite Constellation". *In: International Conference on Localization and GNSS (ICL-GNSS). 2011*.
13. J. Song, G. Xue, and Y. Kang. "A Novel Method for Optimum Global Positioning System Satellite Selection Based on a Modified Genetic Algorithm". *In: PLoS ONE 11.3 (2016).* DOI: 10.1371/journal.pone.0150005.
14. Dan Simon and Hossny El-Sherief. "Navigation satellite selection using neural networks". *In: Neurocomputing 7.3 (1995)*, pp. 247–258.
15. "G1117 VHF DATA EXCHANGE SYSTEM (VDES) OVERVIEW", *IALA-AISM*, 2016, https://www.iala-aism.org/product/vhd-data-exchange-system-vdes-overview-1117/?download=true
16. *IMO Guidelines for Shipborne Position, Navigation and Timing Data Processing. [(accessed on 24February2020)];MSC.1/Circ.1575. 2017*

    Available online: https://docs.imo.org/Category.aspx?cid=106.
17. Gerbeth, Daniel & Caamano, Maria & Circiu, Mihaela-Simona & Felux, Michael, "Satellite selection in the context of an operational GBAS", *NAVIGATION*, 66. 10.1002/navi.284, 2019.
18. W. Shuai, Q. Xing, X. Li, Y. Xing and X. Cai, "An Improved Marine Multi-Constellation Pseudorange Single-Point Positioning Algorithm," *2019 Prognostics and System Health Management Conference (PHM-Paris), Paris, France, 2019*, pp. 1-6, doi: 10.1109/PHM-Paris.2019.00008.
19. Zhang, R., Yao, Y.-. bin ., Hu, Y.-. ming ., and Song, W.-. wei ., "A two-step ionospheric modeling algorithm considering the impact of GLONASS pseudo-range inter-channel biases", *Journal of Geodesy*, vol. 91, no. 12, pp. 1435–1446, 2017. doi:10.1007/s00190-017-1034-x.
20. Sarab Tay, Juliette MARAIS, "Weighting models for GPS Pseudorange observations for land transportation in urban canyons". *6th European Workshop on GNSS Signals and Signal Processing*, *Dec 2013, Germany*. 4p.<hal-00942180>
21. BNC https://igs.bkg.bund.de/ntrip/bnc
22. RTKLib b34c demo5 https://github.com/rtklibexplorer/RTKLIB/tree/demo5
23. PREPaRE SHIPS project weblink: www.prepare-ships.eu